HOT WIRE ANEMOMETER CALIBRATION FOR MEASUREMENTS OF SMALL GAS VELOCITIES

G. E. ANDREWS, D. BRADLEY and G. F. HUNDY

Department of Mechanical Engineering, University of Leeds, Leeds, LS2 9JT, England

Abstract—Hot wires have been calibrated in the régime $0 < Re < 20$ at significant values of Kn, up to 0·12, different temperature loadings, and wire aspect ratios. Calibrations were carried out in a wind tunnel and in a variable pressure rig, in which gas composition could also be varied. The influences of the different variables are shown and discussed. The recommended calibration law is

$$N_c = 0.34 + 0.65\, Re^{0.45}$$

where

$$N_c = \frac{N}{1 - 2\,KnN}$$

for $l/d > 400$, $0.02 < Re < 20$ and with property values taken for the mean gas temperature. The importance of an accurate knowledge of thermal accommodation coefficient at larger values of Kn is discussed.

NOMENCLATURE

$A, B,$ numerical constants;
$a, b,$ numerical constants;
$C_J,$ skin friction coefficient;
$d,$ wire diameter;
$h_s,$ heat transfer coefficient in terms of T_s;
$h_w,$ heat transfer coefficient in terms of T_w;
$I,$ wire current;
$Kn,$ Knudsen number $= L/d$;
$k,$ thermal conductivity of gas;
$k_{T_s},$ thermal conductivity of gas at $T_s[^\circ K]$;
k_{T_m} thermal conductivity of gas at $T_m[^\circ K]$;
$L,$ mean free path of gas molecules;
$l,$ wire length;
$N,$ apparent Nusselt number;
$N_c,$ continuum apparent Nusselt number;
$Nu,$ Nusselt number;
$Nu_c,$ continuum Nusselt number;
$Nu_m,$ measured Nusselt number see equation (6);
$n,$ exponent of Reynolds number;
$Pr,$ Prandtl number;

$Q_C,$ heat loss per second from wire by convection;
$Q_E,$ heat loss per second from wire by conduction to end supports;
$Q_R,$ heat loss per second from wire by radiation;
$q,$ heat flux from wire;
$Re,$ Reynolds number;
$R_w,$ wire resistance;
$r,$ radius;
$T,$ temperature;
$T_g,$ absolute gas temperature remote from wire;
$T_m,$ arithmetic mean of T_g and T_w;
$\bar{T}_s,$ extrapolated wire temperature see equation (8);
$T_w,$ mean wire temperature;
$V,$ voltage;
$v,$ gas velocity;
$W,$ heat loss from wire;
$x,$ power dependence of T for viscosity;

y,   power dependence of T for thermal
      conductivity;

z,   axial development length;

α,   thermal accommodation coefficient;

β,   slip parameter;

γ,   ratio of specific heats;

Δ,   temperature jump distance;

η,   viscosity;

Φ,   see equation (19);

θ,   see equation (11);

ρ,   density of gas;

v,   kinematic viscosity.

1. INTRODUCTION

IN MANY aerodynamic studies, the hot wire anemometer can be calibrated in air under the same conditions of temperature and pressure as the air flow to be studied. However, in some applications this may not be possible. This is particularly true for combustion studies when measurements need to be made for a variety of gaseous mixture compositions and pressures [1]. Under these conditions it is advantageous to have the greatest possible generality in calibration expressions.

The constant temperature hot wire anemometer is suited to the measurement of fluctuating gas velocities because of its small response time and good spatial resolution. In addition to this, it is sensitive to low velocities [2] and can be used to measure velocities below those at which a pitot-static tube may be employed. As the instrument responds to changes in the rate of heat loss from the heated wire element, interpretation of the readings relies on accurate knowledge of the heat transfer mechanism or on the application of an empirical law of calibration. Lack of accurate knowledge of the heat transfer mechanism and of the wire geometry usually means that a calibration for each wire is made under the same conditions of gas temperature and pressure as exist in the experimental situation.

This paper describes the development of, and the results obtained from, a calibration technique which enabled measurements of low, varying gas velocities to be made under conditions where the composition, pressure and temperature of the gas mixture could be different from the gas in which the wire was calibrated. The technique was used to measure the velocity of the unburned gas in closed vessel explosions from which burning velocities of methane–air mixtures were deduced [1]. The gas velocities measured were in the range 50–300 cm.s^{-1} and as a separate wire had to be calibrated for each explosion, the development of a calibration method was important.

Table 1 summarises the wide range of experimental conditions investigated by previous workers, together with those of the present studies.

King [3] was perhaps the first to carry out both a rigorous theoretical treatment and also an accurate experimental investigation of the energy loss from heated platinum anemometer wires. He used a whirling arm in air and showed that the loss was given by

$$W = a + bv^{0.5} \qquad (1)$$

where a and b are constants which depend on the temperature and dimensions of the wire.

The work of subsequent investigators of heat transfer from long cylinders, with flow perpendicular to the axis, was reviewed in 1932 by Ulsamer [41] and in 1954 by McAdams [42]. Table 2 presents the non-dimensional relationships which have been proposed by a number of different workers. One of the major differences between these is the temperature at which the properties of the surrounding gas are evaluated.

The general form of the non-dimensional heat transfer relationship, at fixed Prandtl number, is

$$Nu = A + BRe^n \qquad (2)$$

where A and B are constants. Relationships of this form, when extended over a comparatively large range of Re values, are insufficiently accurate for the more restricted ranges of anemometry studies. This state of affairs, together with observations of deviations from

Table 1. Summary of experimental investigations of heat losses from fine wires

Author	Ref.	Gas	Pressure (at.)	T_w/T_g	$T_g(°K)$	l/d	Velocity calibration range (cm s⁻¹)	K_n	No. wires calibrated
King	3	air	1	1·50-4·50	290	>1000	16-780	<0·01	10
Simmons and Bailey	4	air	1		295	250; 2920	15-3000	0·003	2
Hilpert	5	air	1	1·14-4·47	293	539-5,130	200-3000	<0·005	12
Betchov and Welling	6	air	1	1·25-3·5	293	125-1300	200-1000		11
Ruetenik	7	air	1	1·45-2·12		267	457-1070	0·023	1
Spangenberg	8	air	0·17-2·5	<3	303	200-1300		→0·15	
Cole and Roshko	9	air	1	1·17-1·70	293	460-41000	30-2400	0·008-0·26	6
Sandborn and Laurence	10	air	1-2·72	2	293	100-390	Mach 0·2-0·9	0·001-0·03	3
Beckers et al.	11	air	1	1·003-1·05		1000	~1-20	<0·003	
Hegge Zijnen	12	air	1	1·34		553	19-1615	≪0·01	
Broer et al.	13	air	1	1·14-1·34		695; 1250	5-1800	0·016, 0·03	
Winovitch and Stine	14	air	variable	1·15-1·7		340-712	Mach 0·4-2·5	0·005-0·15	5
Baldwin	15	air	0·1-3·7	1·28-2·0		385	Mach 0·05-0·9	0·0009-0·077	
Collis and Williams	16	air	1	1·1-2·0	300-411	2070-8660	2·5-43, 183-4270	0·001-0·03	4
Webster	17	air	1	1·41, 1·68	283-298	86-1456	378-651	0·003	18
Davies and Fisher	18	air	1	<2	291	400-1200	500-15000	<0·01	
Davis	19	air	1	~2		334-414	500-4500	0·023	7
Parnas	20	air	1	1·2-3·5	293	1000	300-2000	0·0015	1
Almquist and Legath	21	air	1	1·15-1·64		200	3-196	0·020	1
Rasmussen	22	air	1	~2		200	5-260	0·023	1
Zarzycki	23	air	1	1·02-1·8	293-343	200	225-4200	0·020	1
Champagne and Lundberg	24	air	1	1·7	294	200-400	850-3500	0·02	2
Delleur	25	air	1	1·5-1·8		100	100-2100	0·028	1
Champagne et al.	26	air	1	1·7	294	99; 200-600	700-3500	0·02	15
Aihara et al.	27	He-N₂	1-10	1-1·25		>1000	100-300	<0·03	1
Norman	28	air	1	1·68	294	220	100-30000	0·02	1
Ahmad	29, 30	He, N₂ / N₂-CO₂ / He-CO₂	1	0·25-0·75	700-1500			<0·01	1
Davies and Davies	31	air	1	1·7	291	340-590	5-600	0·02	9
Davis and Brunn	32	air	1	1·7	291	200-600	10-10000	0·02	13
Friehe and Schwarz	33	air	1	1·5	293	12-400	600-2100	0·025	6
Bourk et al.	34	CO_2	73-82	1·25-2·0	297-313	141		≪0·01	1
Baccaglini et al.	35	He-N₂ / Ne-N₂	1	1·1		116		0·01	
Bruun	36	air	1	2	291	200-600	50-6000	0·02	1
Hassan and Dent	37	air	1	1·29-3·58	294	small	1200-9000	0·007-0·015	3
Whillier	38	air	1	1·02-1·12	292	small	13-3000	≪0·01	1
Kjellstron and Hedberg	39	air	1	1·6	281-320	220	20-16500	0·02	1
Way and Libby	40	He-air	1	1·17; 2·7	293	150	25-600	0·01-0·03	6
Present work		air; N₂-CH₄	0·24-1	1·05-3·5	298	24-1300	5-5400	0·003-0·12	>100

Table 2. Summary of non-dimensional correlations of heat transfer data from horizontal cylinders in air

Author	Ref.	k, ρ, η at T	Correlation equation	Re
King	3		$Nu = 0.318 + 0.690\ Re^{0.5}$	$0.055 < Re < 55$
Hilpert	5	T_m	$Nu = 0.891\ [Re(T_w/T_g)^{0.25}]^{0.330}$	$1 < Re < 4$
Kramers $\}$	43			
Ulsamer $\int$	41	T_m	$Nu = 0.039 + 0.51\ Re^{0.5}$	$0.01 < Re < 10^4$
Eckert and Soehngen	44		$Nu = 0.43 + 0.48\ Re^{0.5}$	$1 < Re < 4 \times 10^3$
McAdams	42	T_m but T_g for ρ	$Nu = 0.32 + 0.43\ Re^{0.52}$	$0.1 < Re < 10^3$
Scadron and Warshawsky	45	T_g	$Nu = 0.431\ Re^{0.5}$	$250 < Re < 3 \times 10^4$
Churchill and Brier	46	T_g	$Nu = 0.535\ Re^{0.5}\ (T_g/T_w)^{0.12}$	$300 < Re < 2300$
Douglas and Churchill	47	T_m	$Nu = 0.46\ Re^{0.5} + 0.00128\ Re$	$500 < Re < 3 \times 10^5$
Van Der Hegge Zijnen	12	T_m	$Nu = 0.35 + 0.5\ Re^{0.5} + 0.001\ Re$	$0.01 < Re < 10^4$
Glawe and Johnson	48	T_g	$Nu = 0.428\ Re^{0.5}$	$400 < Re < 3 \times 10^3$
Collis and Williams	16	T_m	$Nu\ (T_m/T_g)^{-0.17} = 0.24 + 0.56\ Re^{0.45}$	$0.02 < Re < 44$
Baldwin et al.	49 $\}$	T_g	Graphical correlation	$0.02 < Re < 3 \times 10^5$
Laurence and Sandborn	50 $\int$			$(0.001 < Kn < 37)$
Davies and Fisher	18 $\rfloor$	T_g	$Nu = (2.6/\gamma\pi)\ Re^{0.33}$	$0 < Re < 50$
Richardson	51		$Nu = 0.3737 + 0.37\ Re^{0.5} + 0.056\ Re^{0.66}$	$1 < Re < 10^5$
Parnas	20	T_g	$Nu = 0.823\ Re^{0.5}\ (T_w/T_g)^{0.085}$	$10 < Re < 60$
Ahmad	30	T_m	$Nu(v_g/v_m)^{0.15} = 0.2068 + 0.4966\ Re^{0.45}$	$4 < Re < 40$
Present		T_m	$N_c = 0.34 + 0.65\ Re^{0.45}$	$0.015 < Re < 20$ $l/d > 400$

King's Law, equation (1), led to the experimental investigations of Collis and Williams [16]. They found a change in régime at a Reynolds number of approximately 44, at the onset of vortex shedding, with increase in Re. For values of Re less than this, but greater than 0.02, their results were found to be correlated by the equation

$$Nu \left(\frac{T_m}{T_g}\right)^{-0.17} = 0.24 + 0.56\ Re^{0.45}. \qquad (3)$$

For $44 < Re < 140$ the numerical constants were different and the exponent of Re was 0.51. The temperature ratio in equation (3) was necessary because the evaluation of gas properties at the arithmetic mean film temperature, T_m, did not fully take account of the effect of variation in wire temperature.

Whatever might be the law of convective heat loss from a wire at uniform temperature in a gas of constant properties, this law will be modified by a non-uniform wire temperature, arising from end cooling, and by temperature dependent properties of the wire and gas. Although there is some theoretical basis for the law of heat transfer for uniform conditions having the form

of equation (2) with $n = 0.5$, [3, 52], the question arises whether the law will maintain the same form over a substantial range of Reynolds number, wire aspect ratio, and temperature.

Several workers have computed theoretical temperature distributions along wires with end cooling [3, 18, 19, 26, 31, 53–56] and various correction procedures have been derived [10, 14, 18, 55, 57–59] to allow for conduction loss at the ends, which is principally a function of aspect ratio. However, such corrections are cumbersome to employ and this has not been done in most calibrations. Collis and Williams [16] minimised the end loss by using wires with a length to diameter ratio greater than that commonly encountered in anemometers. Champagne and Lundberg [24] conducted experiments with wires of practical length to diameter ratios of between 200 and 400, giving significant end loss. Their results confirmed the earlier findings of Collis [60] that the main effect of end cooling on the apparent value of Nu, obtained by neglecting the end loss, was to alter the values of numerical constants of equation (3) but not the power of Re.

The theoretical analysis of Davies and Fisher

[18] rests upon the analogy between energy and momentum transfer and gives the expression

$$Nu = \frac{C_j RePr}{\pi\gamma}. \tag{4}$$

It was also proposed that for $Re < 50$

$$C_j = 2{\cdot}6\,Re^{-\frac{3}{4}}. \tag{5}$$

Their results are therefore in the form given by equation (2), but with $A = 0$ and $n = \frac{1}{3}$. The fact that Collis and Williams observed a finite value of A was explained in terms of natural convection and errors of measurement, whilst the value of $n = 0{\cdot}45$ was explained by the heat loss to the supports.

More recently, Davies and Bruun [32], Brunn [36] and Kjellström and Hedberg [39] have used a relationship of the form given by equation (2) in wire calibrations. The constant A is effectively taken to be the Nusselt number corresponding to the heat loss at zero flow and this gives n and B as functions of Re. This is compatible with the linear calibration laws obtained by Collis and Williams [16] and Rasmussen [22] for $Re > 0{\cdot}02$, but in which A, B and n remain unchanged with variation in Re.

Davies and Fisher [18] evaluated fluid properties in equations (4) and (5) at the free stream gas temperature. The experimental values of Nu were less than those given by equations (4) and (5). Hassan and Dent [37] have shown that this deficit increases with an increase in wire temperature and they introduced an empirical temperature loading factor to correct equation (4) for this effect. Hilpert [5], Collis and Williams [16], and Ahmad [29, 30], in his cooled film calibrations, all found it necessary to introduce a small temperature loading factor. Douglas and Churchill [47] found that by evaluating gas properties at T_m the heat transfer data could be correlated for both heat loss and gain from cylinders without such a factor. A similar conclusion has been reached for cooled cylinders by Chevalier et al. [61]. Parnas [20] has calibrated hot wires at various temperature loadings and compared results with gas properties evaluated at T_g, T_w and T_m. He found a strong temperature loading effect when properties were evaluated at T_g, but that when they were evaluated at T_m, this was considerably reduced.

Reference to Table 2 shows that most of the heat transfer equations are either in the general form of equation (2) or in that particular form of this equation in which $A = 0$. Most of those in the latter category are valid for greater Reynolds numbers than are normally employed in hot wire anemometry. For the more usual values of $Re < 40$, the general form of equation (2) is usually valid. However most of the equations in Table 2 apply only for conditions of continuum flow $(Kn < 10^{-3})$.

Davies and Fisher [18] felt that equation (5) was valid for Knudsen numbers less than 0·01 and Reynolds numbers not much less than 10. However, a hot wire of 5 μm dia. at a temperature of 500°K gives a value of Kn of approximately 0·02, evaluated at the mean temperature, in air at atmospheric pressure and temperature. Consequently non-continuum effects must always be considered in anemometer wire calibrations. Davies and Fisher's analysis uses the continuum drag coefficient measurements of Tritton [62]; however, several workers [63–68] have investigated this coefficient in non-continuum flow and found the values to be dependent upon Kn. Couderville et al. [66] found that the coefficient decreased markedly with an increase in Kn for $Re < 10$.

Several workers [8, 15, 16, 49, 69] have shown from hot wire calibrations, that there is a decrease of Nu with an increase in Kn. This arises from the change in physical régime from continuum flow to slip flow, when the mean free path of the gas molecules becomes comparable to the wire diameter. Under these conditions, the temperature adjacent to the wire is lower, and the gas velocity higher, than continuum theory would suggest. Collis and Williams [16] found discrepancies when using wires of different diameter, owing to changes in the value of Kn. The range of values of Kn in their work was too

small to attempt an accurate evaluation of the effect of temperature jump on heat transfer. A simplified analyses gave rise to the following expression, applicable at low temperature loadings, which corrects the measured value, Nu_m, to the continuum Nusselt number, Nu_c.

$$Nu_c = \frac{Nu_m}{1 - 2KnNu_m}. \tag{6}$$

Even within what is normally considered to be the continuum regime with $Kn < 10^{-3}$, slip effects can occur at low values of Re if the thermal accommodation coefficient for the gas on the wire is small, $(\alpha \ll 1)$. The effect of the thermal accommodation coefficient on wire calibrations has been predicted theoretically by Levey [70] and Kassoy [71]. It has also been demonstrated experimentally by Kassoy et al. [27, 35] for hot wires in nitrogen, helium and neon.

The present work was carried out to obtain more information of the calibration laws for hot wire anemometer probes in the régime $0 < Re < 20$, at significant values of Kn, and for different temperature loadings and wire aspect ratios. A clarification of these laws enabled calibrations to be conveniently carried out under conditions of gaseous composition and pressure different from those in the explosion studies.

2. NON-CONTINUUM HEAT TRANSFER

In hot wire anemometry the use of wire diameters of 5 μm and less means that non-continuum effects cannot be ignored, even at a pressure of one atmosphere. Effectively this means that hot wires operate in the transitional regime of rarefied gas dynamics, between free molecular and continuum flows. There are extensive data on heat transfer and drag for cylinders in this régime [72–75]. Heat transfer from cylinders has been examined for two different circumstances; low density high speed flow [8, 10, 15, 45, 49, 57, 59, 69, 75] and low density static gases [76–79]. For both sets of conditions most investigators have been primarily interested in the effect of Knudsen number. But this alone is an insufficient parameter to allow for the effects of significant mean free paths. In addition, the effect of the thermal accommodation coefficient, α, must be considered [80].

This coefficient may be defined [81], as the fractional extent to which those molecules which fall on the surface, and are reflected or re-emitted, have their mean energy adjusted or "accommodated" towards that value which would obtain if the returning molecules were to issue as a stream from a mass of gas at the wall temperature. It is expressed quantitatively by

$$\alpha = \frac{E_i - E_r}{E_i - E_w} \tag{7}$$

where E_i and E_r are respectively the energy fluxes incident and re-emitted from a differential surface element. The quantity E_w is the energy flux away from the surface if all incident molecules were to be re-emitted with a Maxwellian velocity distribution at the surface temperature.

The temperature jump at a heated surface in a rarefied gas is given by [81]

$$T_w - T_s = -\Delta \frac{\partial T}{\partial r} \tag{8}$$

where T_w is the wire temperature and T_s would be the gas temperature if the temperature gradient remained unchanged up to the surface. Kennard [81] has shown that the temperature jump distance, Δ, is related to the thermal accommodation coefficient by

$$\Delta = \frac{2 - \alpha}{\alpha} \frac{2\gamma}{\gamma + 1} \cdot \frac{L}{Pr}. \tag{9}$$

For most gases and surfaces Δ is of the same order as the mean free path, L. A slip parameter, β, is also used [69, 71], given by

$$\beta = \frac{\Delta}{d} = \theta Kn \tag{10}$$

where

$$\theta = \frac{2 - \alpha}{\alpha} \frac{2\gamma}{\gamma + 1} \frac{1}{Pr}. \tag{11}$$

Diessler [82] has shown that equation (9) is essentially a first order approximation, which is valid if the velocity and temperature profiles are linear over a distance of a mean free path. At high values of Kn the profiles will be non linear, and this state of affairs significantly increases the temperature jump distance. In the present work the values of Kn are such as to justify the use of equation (9) without significant error.

Heat transfer coefficients may be defined either in terms of T_w or of T_s. If q is the heat flux from the wire then

$$q = h_w(T_w - T_g) = h_s(T_s - T_g) \qquad (12)$$

and

$$q = -k_{T_s}\left(\frac{\partial T}{\partial r}\right)_{T_s}. \qquad (13)$$

From equation (12)

$$h_s = h_w\left(1 + \frac{T_w - T_s}{T_s - T_g}\right). \qquad (14)$$

From equations (8), (12) and (13) and the fact that Δ is of the same order as the mean free path, we can write

$$\frac{T_w - T_s}{T_s - T_g} = \frac{\Delta_{T_s} h_s}{k_{T_s}}. \qquad (15)$$

The Nusselt number, Nu, and the continuum Nusselt number, Nu_c, are defined by $Nu = h_w d/k_{T_m}$ and $Nu_s = h_c d/k_{T_m}$. Equations (14) and (15) yield

$$Nu_c = \frac{Nu}{1 - Nu\Delta_{T_s}k_{T_m}/k_{T_s}d}. \qquad (16)$$

From equations (10) and (16)

$$Nu_c = \frac{Nu}{1 - Nu\theta_{T_s}Kn_{T_s}k_{T_m}/k_{T_s}}. \qquad (17)$$

For a perfect gas at constant pressure Kn $\alpha\ T^{0.5+x}$, where viscosity is proportional to the absolute temperature raised to the power x. If k is proportional to T^y then

$$Nu_c = \frac{Nu}{1 - \phi KnNu} \qquad (18)$$

where Kn is evaluated at T_m and

$$\phi = \theta_{T_s}\left(\frac{T_s}{T_m}\right)^{0.5+x-y}. \qquad (19)$$

If T_s is close to T_w then

$$\phi = \theta_{T_s}\left|\frac{2T_w/T_g}{1 + T_w/T_g}\right|^{0.5+x-y}. \qquad (20)$$

Most slip flow heat transfer relationships can be expressed by an equation of the type of equation (18). The factor ϕ is always dependent upon the thermal accommodation coefficient but varies with the method of analysis and the geometrical configuration. The analysis of Kassoy [71] for a hot wire gives an equation of the same form as equation (18), but with ϕ as a more complex function of T_w/T_g and β. Mikami et al. [83] have shown that the relationship between Nusselt number, Kn and α is the same for both a sphere and a cylinder. For a sphere, Kavanau [84] obtained a relationship, for $(T_w - T_g)/T_g \ll 1$, which gives $\phi = \theta$ in equation (18). He showed that this equation gives agreement with the theory of Sauer and Drake [80] for a cylinder.

The major problem in the use of equation (18) is the determination of the correct value of thermal accommodation coefficient. A number of reviews [85–91] indicate that there is wide disagreement on the value of α for a given metal–gas interface, on the pressure and temperature dependence of α, on the best experimental method for its determination, and on the theoretical derivation of α. The value of α is strongly dependent upon the character of the wire surface, varying by a factor of ten in the case of helium between a completely out-gassed wire and one with absorbed gas [86, 88]. The application of thermal accommodation coefficient data, obtained on clean wires, to contaminated anemometry wires may not be justifiable. This has led to the direct determination of α from hot wire calibrations by the use of equation (18), [27, 35, 69, 92]. Such results

Table 3. Selected comparison of accommodation coefficients of various gases

Incident gas	Surface	Surface condition	$T_w(°K)$	$T_g(°K)$	Accommodation coefficient α	Author	Ref.
N_2	Tungsten	Gas covered	1870	293	0·087	De Poorter and Searly	93
N_2	Tungsten	Gas covered	373·1	294	0·843	Ubisch	94
N_2	Tungsten	Gas covered			0·868	Amdur and Guildner	95
N_2	Platinum	Gas covered			0·816	Amdur and Guildner	95
N_2	Platinum	Nitrogen covered	297	277	0·89	Dickens	97
N_2	Tungsten	Nitrogen covered	403	304	0·60	Wachman	96
O_2	Tungsten	Gas covered	373·1	294	0·887	Ubisch	94
O_2	Tungsten	Gas covered			0·905	Amdur and Guildner	95
O_2	Platinum	Gas covered			0·853	Amdur and Guildner	95
O_2	Platinum	Gas covered	297	277	0·90	Dickens	97
air	Platinum	Gas covered	297	277	0·90	Dickens	97
air	Platinum	Gas covered		280	0·75	Taylor and Johnston	100
air	Aluminium	Gas covered			0·87–0·95	Wiedmann and Trumpler	99
air	Platinum	Hot wire probe			0·85	Atassi and Brun	69
air	Platinum	Hot wire probe			0·85	Schmidt and Cresci	92
CH_4	Platinum	Gas covered		300	0·78	Grilly et al.	98
C_2H_6	Platinum	Gas covered	293–370		0·97–0·77	Schäfer	101
H_2	Tungsten	Gas covered	298		0·357	Amdur and Guildner	95
H_2	Platinum	Gas covered	291	271	0·36	Dickens	97
CO_2	Platinum	Gas covered	324·4	303·1	0·786	Thomas and Golicke	102
He	Tungsten	Clean		331	0·0186	Roach and Thomas	103
He	Tungsten	Gas covered			0·470	Amdur and Guildner	95
He	Tungsten	Hot wire probe			0·04	Baccaglini et al.	35
Ne	Tungsten	Clean		339	0·0428	Roach and Thomas	103
Ne	Tungsten	Hot wire probe			0·093	Baccaglini et al.	35

are compared with those of the more classical methods for several gases, in Table 3.

In equation (11) the thermal accommodation coefficient is associated with temperature T_s, which is usually close to the wire temperature T_w. However, the effect of wire temperature upon α is not clear. Grilly et al. [98] found that α decreases as T_w increases, de Poorter and Searly [93] indicate very low values of α at high wire temperatures, whilst Oliver and Farbor [104] have found that α is also a function of $(T_w - T_g)$. The theory of Goodman and Wachman [105] shows that α can either increase or decrease with temperature, depending upon the gas, and this is in good agreement with experimental results [103, 105]. There is similar uncertainty about the effect of pressure on α [89, 96].

For the present work, involving nitrogen, oxygen, air and methane. platinum and tungsten. reference to Table 3 suggests a value for α of 0·85. Equation (11) then gives a value of

$\theta_{T_s} = 2·23$. Corresponding values of ϕ are obtained from equation (20). With values of x and y of 0·68 and 0·80 respectively, derived from the properties of air listed in [106], this latter equation gives $2·23 < \phi < 2·67$ corresponding to $1 < T_w/T_g < 3·5$, respectively. Error analysis based upon equation (18) shows that a 20 per cent error in ϕ leads to a 1·27 per cent error in Nu_c for $Nu = 1$, $Kn = 0·03$ and $\phi = 2$. For accurate work at high values of Knudsen number there is a need for more accurate values of α than exist at present.

In the present work a constant value of 2·0 has been assigned to ϕ. This is sufficiently accurate and reduces equation (18) to equation (6), first used by Collis and Williams for low temperature loadings.

3. EXPERIMENTAL

A DISA constant temperature anemometer, type 55 A01. was used. Standard DISA probes.

types 55 A25 and 55 F31, supported the wires, which were welded to the probes using a spot welding micromanipulator. Wires of platinum and tungsten, of diameter 1–25 μm and of various length were used; the aspect ratio, l/d, ranged from 24 to 1300. Wire lengths between the spot welds were measured to an accuracy of ±0·001 in., using a travelling microscope. Wire diameter values were taken as nominal, as facilities for the accurate measurement of wire diameters of less than 5 μm were not available. A check on this assumption was made by calculating the average wire diameter from the measurement of its length and electrical resistance. Thirty wires gave an average diameter of 3·817 μm compared with a nominal diameter of 3·810 μm. Davis's [19] measurements of wire diameter using an electron microscope showed that the diameter differed by less than 2 per cent from the nominal value of 5 μm. Thus the use of nominal wire diameters was adequate. The mean wire temperature was deduced from the temperature–resistance calibration for platinum given by Wise and Vines [107]. By varying the operating wire resistance the mean wire temperature was varied between 310°K and 1050°K.

Experiments were carried out on two rigs. A closed circuit wind tunnel, with a horizontal 3 ft × 2 ft low turbulence uniform velocity working section, was used for air calibration at atmospheric pressure in the air velocity range of 200–5400 cm s^{-1}. Wire voltages were calibrated against a pitot static tube, the head of water being read to an accuracy of 0·001 in. on a micromanometer.

A variable pressure low velocity facility was used for calibrations over the pressure range between 0·24 and 1 atmosphere and the velocity range between 5 and 250 cm s^{-1}. This rig consisted of 2 m long vertical tube built up from a 0·76 m long 25.4 mm bore tube opening into a 50·8 mm bore tube. The flow development length in a circular tube is given by [108, 109]

$$z = 0\cdot057\, r\, Re. \qquad (21)$$

Application of this equation gives maximum permissible centre line velocities, for fully developed flow, of approximately twice those used in the calibration. Thus, the flow was always fully developed at the wire and the centre line velocity could be taken as twice the mean velocity. Either air or a prepared gas mixture was drawn up to the tube via a buffer tank of 0·5 m^3 capacity to a vacuum pump. The volume rate of flow was measured using a gas meter. Gas temperatures were always equal to the ambient temperature.

In both rigs the wire was mounted normal to the flow. This adjustment was made with sufficient accuracy by eye. The wire probe was mounted horizontally, but the vertical flow in the low pressure rig could influence the results obtained at very low velocity, where free convection effects are important. However, as only the approximate Reynolds number at which free convection effects become important was required in this work, the effect of wire orientation in the mixed free and forced convection régime was considered unimportant. Outside this régime Rasmussen [22] and Davis and Davies [31] have shown that wire orientation has no influence on the heat transfer.

For the Nusselt number to be evaluated, the heat loss by convection from the wire must be known. The electrical energy supplied will be dissipated by convection, radiation and conduction to the supporting leads. Because the wire is of finite length, this can be expected to result in a temperature variation along the wire. The energy equation for the wire may be expressed thus:

$$I^2 R_w = Q_C + Q_R + Q_E = q\pi dl \qquad (22)$$

where R_w is the total wire resistance. Owing to the temperature variation along the wire resulting from the cooling by the main leads, the resistivity and hence the energy supplied per unit length will vary along the wire. The value of Q_R may be found. In the present work it was no more than 0·25 per cent of the electrical

heating. Hence, heat was considered to be dissipated by conduction to the end supports and convection only.

An accurate estimation of the conduction heat loss involves a knowledge of the temperature gradient of the wire at the supports, and to find this a computation of the temperature distribution along the wire is necessary. If this heat loss is significant, it must be allowed for if the true value of Nu is to be found. The length to diameter ratio is important in this respect, since the larger its value, the smaller the proportion of heat conducted to the supports.

For calibration purposes, an absolute value of Nusselt number is not required; all that is needed is a relationship which will hold under the experimental conditions. Hence, unless the change in the value of the heat loss to the supports as the Reynolds number changes can be accurately predicted, a correction for this heat loss will be of no assistance. It was therefore decided to correlate Reynolds number with an apparent Nusselt number, N, which was calculated without allowance for the conduction heat loss and on the basis of a temperature difference of $(T_w - T_g)$, with a value of T_w derived from the mean wire resistance.

A corresponding value of continuum apparent Nusselt number is derived from N by an adaptation of equation (18), with $\phi = 2.0$,

$$N_C = \frac{N}{1 - 2KnN}. \tag{23}$$

The results of the wire calibrations are presented in a form given by an adaptation of equation (2), namely,

$$N_C = A + BRe^n. \tag{24}$$

In equations (23) and (24) all temperature dependent properties are evaluated at the mean temperature, T_m.

Error analysis [110] of the effects of experimental errors on the derived values of Re and N shows maximum errors of ± 3.4 per cent and ± 10.7 per cent respectively, on these parameters, for a wire of 2.54 mm length and a

temperature ratio, T_w/T_g, of 2.0. The major source of errors for both dimensionless numbers lies in the measurement of wire resistance, with its associated errors in T_w and consequently, in values of gas properties. The analysis also shows that the experimental scatter for a given wire at this temperature ratio is ± 0.5 per cent for Re and ± 2 per cent for N. The former figure largely derives from errors in measurement of gas velocity and the latter from errors in voltage measurement. The experimental results accord with the findings of the error analysis.

4. RESULTS AND DISCUSSION

4.1 Power law and low velocity calibration

The present work shows that Nusselt and Reynolds numbers are best correlated by a relationship of the form of equation (24) and in which $n = 0.45$, as in the investigations of Collis and Williams [16]. This is shown to be valid for $0.02 < Re < 22$, $1.05 < T_w/T_g < 3.5$, $0 < Kn < 0.1$, and $24 < l/d < 1300$. Other values of n have been proposed, but in the present work a value of $n = 0.45$ gave the best straight line. Figure 1 shows values of N_C obtained with the same wire at low Reynolds numbers in both the variable pressure rig and also, at higher Reynolds numbers, in the wind tunnel. This reveals the same calibration law for both rigs.

The results of Collis and Williams [16] were for large aspect ratios. The present and other work [24, 33] show that the power law, with $n = 0.45$, can be used for the lower values of l/d, common in hot wire anemometry. Ahmad [29, 30] has shown that the same value of n holds in cooled film anemometry, where the gas is hotter than the wire. There appears to be no advantage in imposing a value of A in equation (24) which is equal to the Nusselt number at zero flow, as proposed by some workers [32, 36, 39]. This would create the inconvenience of B in equation (24) and the exponent n becoming functions of Re.

Accurate calibration of hot wires at low velocity (< 200 cm s^{-1}) is difficult, as standard pitot tubes cannot be used. The calibration can
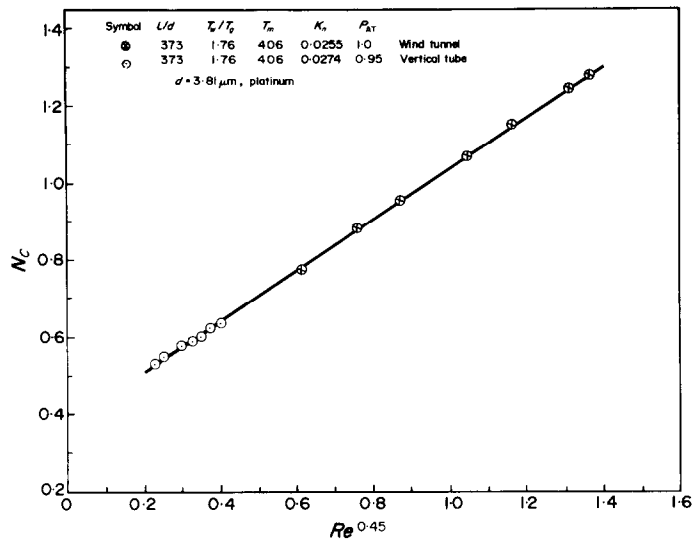
FIG. 1. Low velocity calibration in variable pressure rig and wind tunnel.
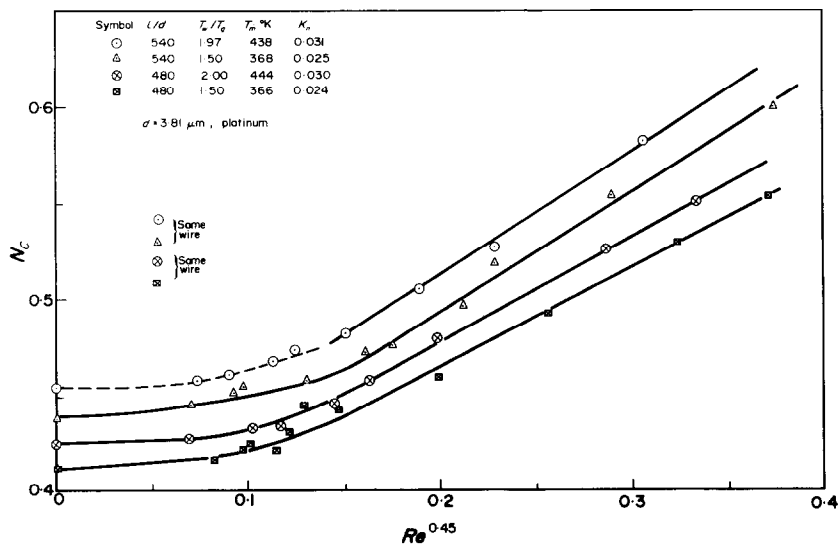

FIG. 2. Low velocity calibration in variable pressure rig.

be achieved by moving the hot wire relative to the stationary fluid either on a whirling arm [3, 13] or in a towing tank [38, 111, 112]. The alternative method, used in this and other work [22, 27, 34, 35], is to place the hot wire at the centre of a long tube in which fully developed flow has been established. Difficulties are involved in the measurement of low flow rates [113], which can be overcome by expelling air from a container using water [21, 27, 35, 114]. Collis and Williams [16] have calibrated their wires against a Simmon's shielded anemometer, whereas Cole and Roshko [9] have utilised the frequency of vortex shedding from a rod. There have been several theoretical analyses of the velocity field and heat transfer from cylinders with cross flow for values of Reynolds number less than unity [7, 70, 71, 115–124]. These use either the Oseen approximation or the method of matched asymptotic expansions of Proudman and Pearson [115]. Results are in reasonable agreement with experiments over the range $0.02 < Re < 0.3$ [120]. The analyses are of limited usefulness as most of the results [7, 115–122] are only applicable to infinite cylinders with no slip flow. The analysis of Yamamota

[124] for slightly rarefied gases, which includes second order Knudsen number effects, shows considerable differences from experimental measurements [69] due to a neglect of the effect of thermal accommodation coefficient.

Results obtained in the variable pressure rig and shown in Fig. 2 indicate that the straight line calibration is valid for $Re > 0.015$, for wire diameters < 4 μm. This is a slightly lower limit than that found by Collis and Williams [16] but is in agreement with the mixed convection results of Gebhart et al. [125]. Below this value of Reynolds number Grashof number effects are significant. Figure 2 also shows that extrapolation of the wind tunnel calibration down to this limiting Reynolds number is a useful method of low velocity calibration.

4.2 Effects of wire temperature, gas pressure, and composition

The results of calibrations in air at atmospheric pressure for values of temperature ratio, T_w/T_g, ranging from 1.18 to 3.56 are shown in Fig. 3, for the two aspect ratios of 320 and 1200. It is seen that by evaluating gas properties at the temperature T_m, equation (24) is valid over a
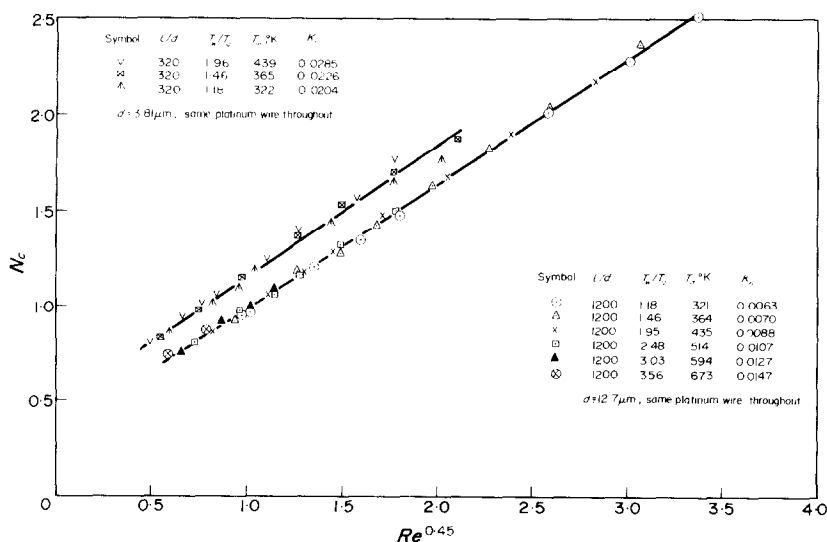

FIG. 3. Calibration at different values of T_w/T_g.

wide range of values of T_m. No dimensionless temperature loading factor was necessary in this equation.

The influences of gas pressure were investigated over the range 0·238–1·0 atmosphere in the variable pressure rig. The corresponding range of Knudsen numbers was 0·016–0·094 and

the calibrations for two dissimilar wires are shown in Fig. 4. The use of equation (23) to allow for slip flow effects leads to good correlations by equation (24). The figure shows the extrapolations of wind tunnel calibrations obtained at higher Reynolds numbers for each wire. The variable pressure rig experimental
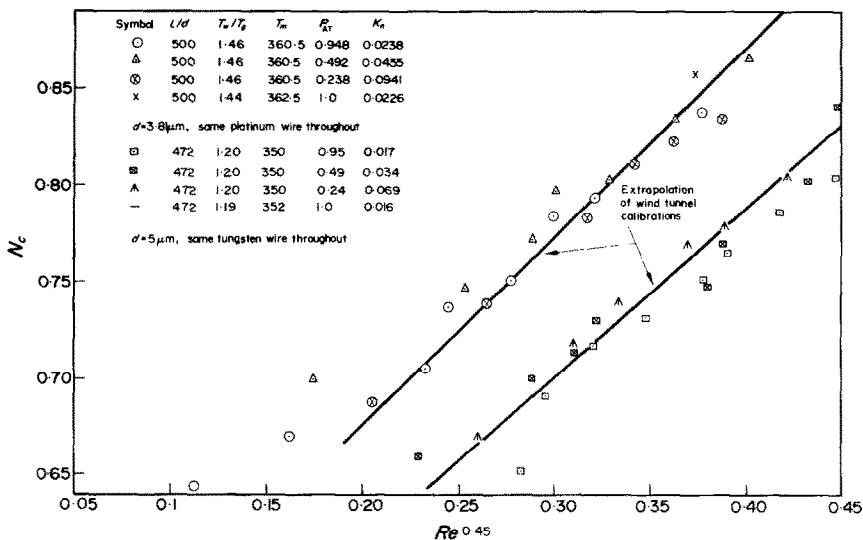
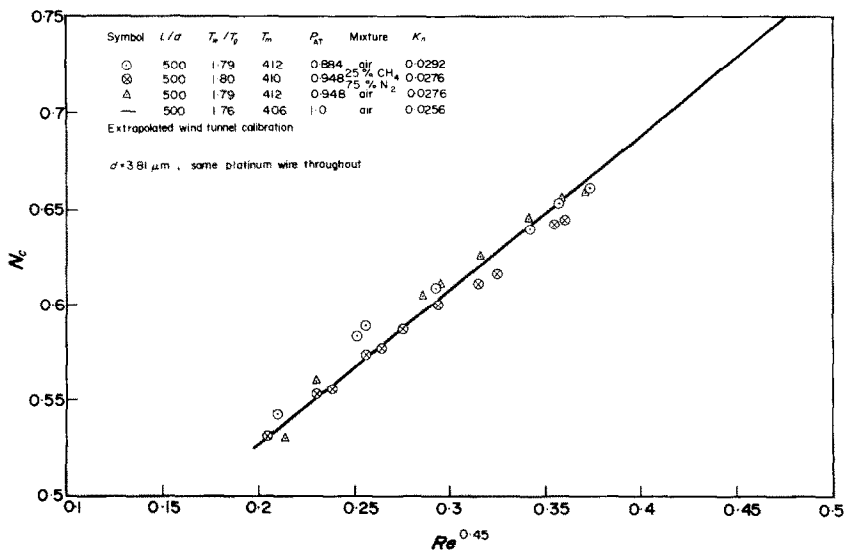FIG. 4. Calibration at different values of Knudsen number.

FIG. 5. Calibrations in air and methane–nitrogen.

points lie quite satisfactorily with regard to the two straight lines but, as in Fig. 2, the increased experimental scatter at low gas flows is clear.

The variable pressure rig was also used for calibrating wires in gases of different composition. Calibrations of a wire in both air and a mixture of 25% methane and 75% nitrogen are shown to be in good agreement in Fig. 5. Extrapolation of a wind tunnel calibration at higher Reynolds numbers and with the same wire is shown by the full line. This shows good correlation between results from the two rigs. Data for the viscosity and thermal conductivity of methane were taken from Svehla [126] and from International Critical Tables [127] for air. Values for the viscosity of the gas mixture

were computed from the formula given by Wilke [128] and for the thermal conductivity formula given by Mason and Saxena [129].

4.3 Effects of wire aspect ratio

In anemometry considerations of spatial resolution require that the lengths of wires be short. Most hot wires operate in the range of aspect ratio $100 < l/d < 600$, but Table 1 shows that few investigations have been published of low velocity heat transfer characteristics within this range. The investigations of King [3] and Collis and Williams [16] were for large aspect ratios, outside the practical range of interest. Collis and Williams [16] suggested that end conduction losses could be ignored for aspect
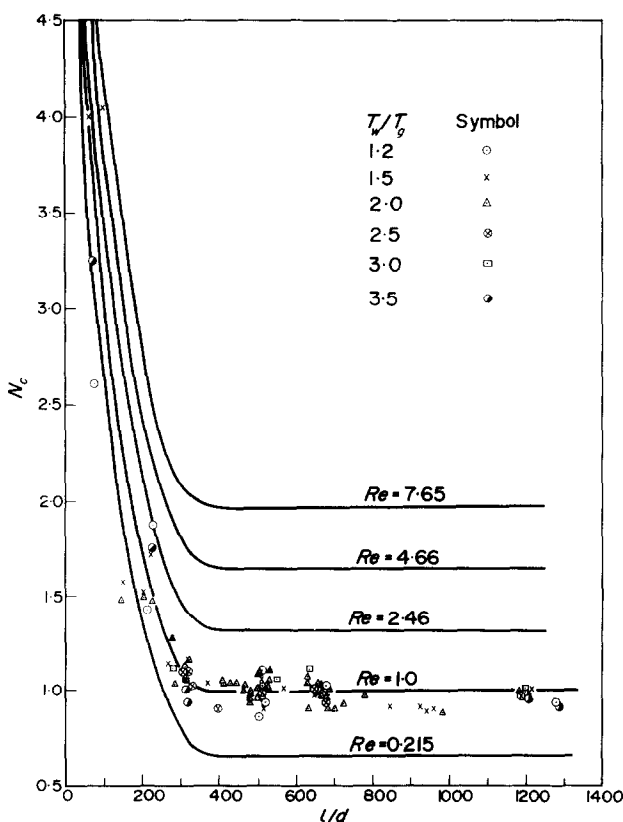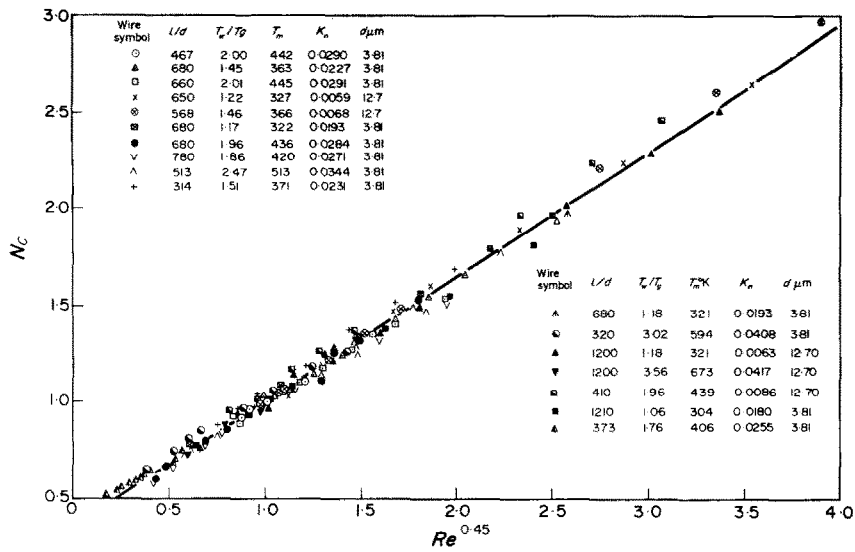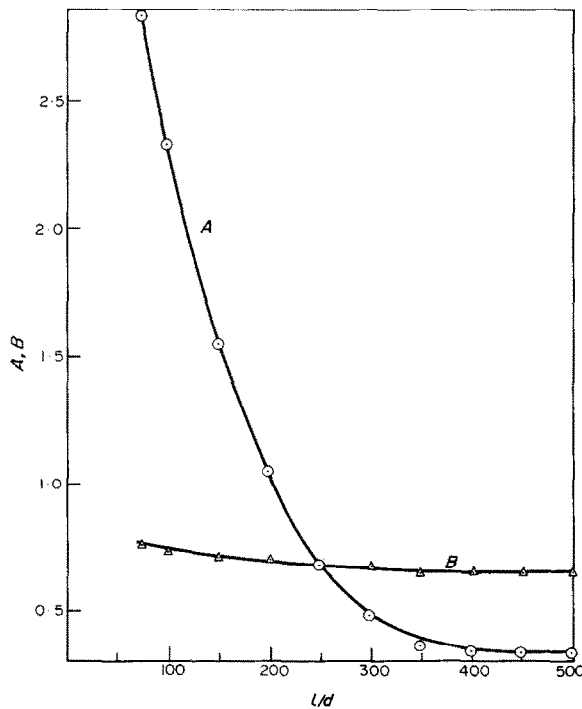

FIG. 6. Effect of aspect ratio at different Reynolds numbers and wire temperatures.

Wire symbol	l/d	T_w/T_g	T_m	K_n	$d\,\mu m$
⊙	467	2·00	442	0·0290	3·81
▲	680	1·45	363	0·0227	3·81
◻	660	2·01	445	0·0291	3·81
×	650	1·22	327	0·0059	12·7
⊗	568	1·46	366	0·0068	12·7
⊞	680	1·17	322	0·0193	3·81
●	680	1·96	436	0·0284	3·81
∨	780	1·86	420	0·0271	3·81
∧	513	2·47	513	0·0344	3·81
+	314	1·51	371	0·0231	3·81

Wire symbol	l/d	T_w/T_g	$T_m°K$	K_n	$d\,\mu m$
▲	680	1·18	321	0·0193	3·81
◉	320	3·02	594	0·0408	3·81
▲	1200	1·18	321	0·0063	12·70
▼	1200	3·56	673	0·0417	12·70
◙	410	1·96	439	0·0086	12·70
■	1210	1·06	304	0·0180	3·81
▲	373	1·76	406	0·0255	3·81

FIG. 7. Calibration for a variety of conditions with $314 < l/d > 1210$.

FIG. 8. Variation, with aspect ratio, of A and B in equation (24), with $n = 0.45$.

ratios greater than 2000 but Gebhart and Pera [130] found that conduction was significant in the range $1000 < l/d < 16\,000$.

The variation of continuum apparent Nusselt number, N_c, with aspect ratio is shown in Fig. 6, for different values of Reynolds number. These curves have been derived from the calibration of over 100 wires, covering a wide range of temperature ratios and Knudsen numbers. For clarity the individual points have only been plotted for $Re = 1.0$. Figure 6 shows that at values of aspect ratio between 300 and 400 there is a sharp rise in N_c as Re is reduced. Variations in end effects with aspect ratio seem

In Section 1, reference was made to computations of theoretical temperature distributions along a hot wire. These show how a decrease in aspect ratio results in an increase in the value of the apparent Nusselt number, as a consequence of an increase in the conduction end loss. This loss is proportionately greater for a given wire the lower the Reynolds number. The variation of temperature along the wire affects not only the rate of heat transfer but also the values of gas properties. The effects of variation in aspect ratio upon the values A and B in equation (24) are shown in Fig. 8, which has been derived from Fig. 6. The principal effect
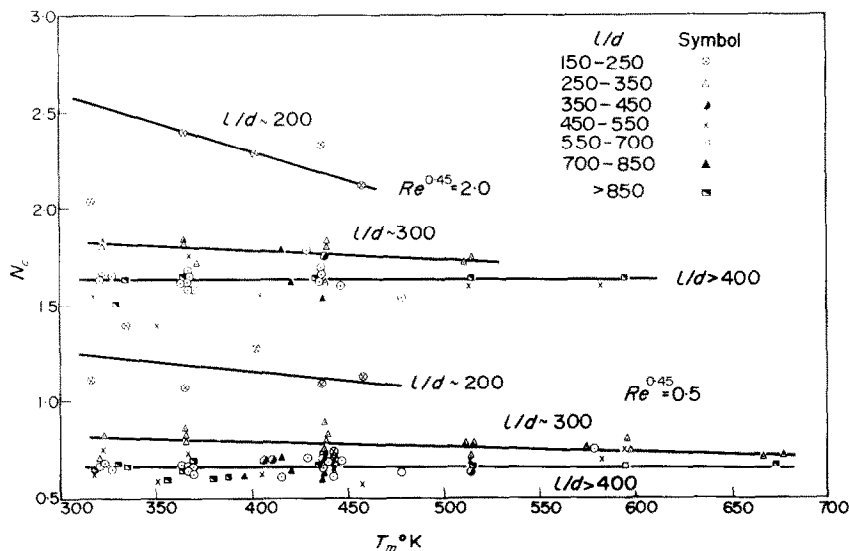

FIG. 9. Effect of temperature for different aspect ratios.

to be unimportant when the latter is greater than 400. For this condition it was found that the results are best correlated by

$$N_c = 0.34 + 0.65\,Re^{0.45}. \qquad (25)$$

This relationship is derived from the horizontal portions of the curves in Fig. 6 and corresponds closely to the line in Fig. 7, correlating the calibration of several wires for various conditions over the range $314 < l/d < 1210$.

of a decrease in aspect ratio is an increase in A, the exponent n being unchanged and the gradient B only slightly affected. Cole and Roshko [9] have previously noted this effect and curves such as those shown in Fig. 6 have been predicted theoretically by Betchov and Welling [6]. Their theoretical and experimental results indicate that end conduction may be neglected if $l/d > 400$.

Any effect of wire temperature upon the

calibration law is shown in Fig. 9, where N_c is plotted against T_m for two different values of Re. The relationships show a negligible temperature dependence of N_c for aspect ratios greater than 400. Many hot wire anemometers have aspect ratios of approximately 200. The present work suggests that for consistency of calibration a value of 400, or greater, is preferable.

4.4 Effects of Knudsen number

The present work shows the importance of allowing for slip flow effects in equation (25). The percentage error, $100(N_c - N)/N$, in neglecting Knudsen number effects is shown for a range of conditions in Fig. 10. The curves show
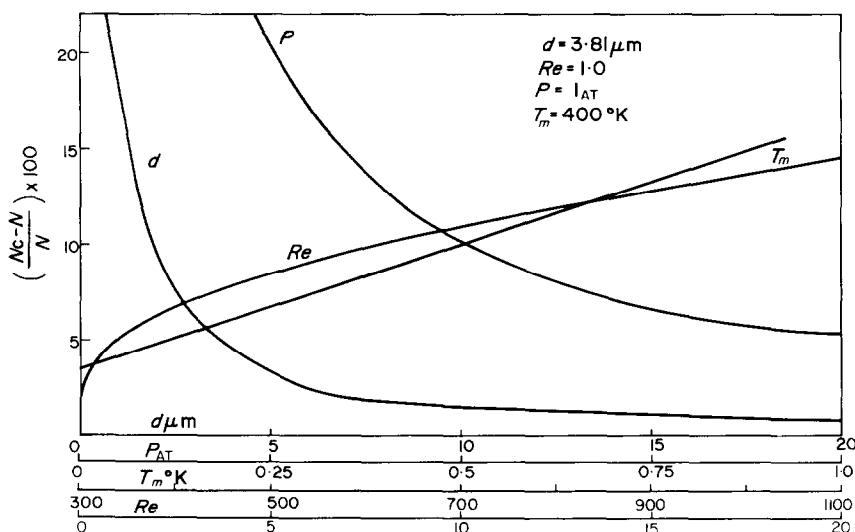
5. CONCLUSIONS

(1) Hot wire anemometers usually operate under slip flow conditions in which $Kn > 0.001$. There is a need for more accurate values of thermal accommodation coefficient for use at higher values of Knudsen number.

(2) The continuum apparent Nusselt number is given by

$$N_c = \frac{N}{1 - \Phi\, Kn\, N}.$$

With $\Phi = 2$, for $0.02 < Re < 20$, $24 < l/d < 1300$, $300 < T_m < 675°K$ and $0.008 < Kn < 0.1$,

10. Percentage error in neglecting Knudsen number effects.

the effect of variation in Reynolds number, wire diameter, wire temperature and air pressure and are constructed from equations (23) and (25), by varying each parameter separately keeping all others constant at the values shown. For normal hot wire operating conditions the error caused by neglect of slip flow conditions will vary between 5 and 15 per cent. These errors account for the diameter effect noted by Cole and Roshko [9], at fixed aspect ratio.

the calibration law is of the form

$$N_c = A + B\, Re^{0.45}$$

with property values taken for the mean gas temperature.

(3) When the aspect ratio is greater than 400, end cooling effects are not severe and the calibration law is

$$N_c = 0.34 + 0.65\, Re^{0.45}.$$

REFERENCES

1. D. BRADLEY and G. F. HUNDY, Burning velocities of methane–air mixtures using hot wire anemometers in closed vessel explosions. Thirteenth Symposium (International) on Combustion, 575–583 The Combustion Institute, Pittsburgh (1971).
2. DISA, Constant Temperature Anemometer Type 55AO1, General Description, pp. 12–13 (1966).
3. L. V. KING, On the convective heat transfer from small cylinders in a stream of fluid. Determination of convective constants of small platinum wires with application to hot-wire anemometry. *Phil. Trans. R. Soc.* **214A**, 373–432 (1914).
4. L. F. G. SIMMONS and A. BAILEY, A hot wire instrument for measuring speed and direction of air flow. *Phil. Mag.*, Ser. 7, **3**, 81–96 (1927).
5. R. HILPERT, Wärmeabgabe von geheizten Drähten und Rohren in Luftstrom. *Forsch. Geb. Ing.* **4**, 215–224 (1933).
6. R. BETCHOV and W. WELLING, Some experiences regarding the nonlinearity of hot wires, *Verh. Kon. Ned. Akad. Wet.* **52**, 195–207 (1949). English Translation: NACA TM 1223 (1952).
7. J. R. REUTENIK, The effect of the temperature dependence of King's "Constant" A on the hot wire sensitivity coefficient. *J. Aeronaut. Sci.* **23**, 502–503 (1955).
8. W. G. SPANGENBERG, Heat loss characteristics of hot wire anemometers at various densities in transonic and supersonic flow, NACA TN 3381. National Bureau of Standards, Washington (1955).
9. J. COLE and A. ROSHKO, Heat transfer from wires at Reynolds numbers in the Oseen range, Proc. Heat Transfer and Fluid Mech. Inst., Univ. California–Berkeley, pp. 13–23 (1954).
10. V. A. SANDBORN and J. C. LAURENCE, Heat loss from yawed hot wires at subsonic Mach numbers. NACA TN 3563 (1955).
11. H. L. BECKERS, L. W. TER HAAR, L. T. TJOAN, H. J. MERK, J. A. PRINS and J. SCHENK, Heat transfer at very low Grashof and Reynolds numbers, *Appl. Sci. Res.* **A6**, 82–84 (1956).
12. B. G. VAN DER HEGGE ZIJNEN, Modified correlation formulae for the heat transfer by natural and forced convection from horizontal cylinders, *Appl. Sci. Res.* **A6**, 129–140 (1956).
13. L. J. F. BROER, C. J. HOOGENDOORN, and A. KORTLEVEN, Some investigations on low speed anemometry. *Appl. Sci. Res.* **A7**, 1–13 (1957).
14. W. WINOVICH and H. A. STINE, Measurements of the nonlinear variation with temperature of heat-transfer rate from hot wires in transonic and supersonic flow, NACA TN 3965 (1957).
15. L. V. BALDWIN. Slip-flow heat transfer from cylinders in subsonic air streams. NACA TN 4369 (1958).
16. D. C. COLLIS and M. J. WILLIAMS, Two-dimensional convection from heated wires at low Reynolds numbers. *J. Fluid Mech.* **6**, 357–384 (1959).
17. C. A. G. WEBSTER, A note on the sensitivity to yaw of a hot wire anemometer. *J. Fluid Mech.* **13**, 307–312 (1962).
18. P. O. A. L. DAVIES and M. J. FISHER, Heat transfer from electrically heated cylinders. *Proc. R. Soc.* **280A**, 486–527 (1964).
19. M. R. DAVIS, Convective heat transfer from yawed hot wires. Univ. Southampton, ISAV. Mem. No. 188 (1964).
20. A. L. PARNAS, Temperature loading influence on heat transfer from a cylinder in an air cross flow. *Inzh.-Fiz. Zh.* **7**, 37–44 (1964).
21. P. ALMQUIST and E. LEGATH, The hot-wire anemometer at low air velocities. DISA Information No. 2, 3–4 (1965).
22. C. G. RASMUSSEN, The air jet hot-wire microphone. DISA Information No. 2, 5–13 (1965).
23. R. ZARZYCKI, Measuring gas-dynamic parameters, DISA Information No. 4, 30–35 (1966).
24. F. H. CHAMPAGNE and J. L. LUNDBERG, Linearizer for constant temperature hot-wire anemometer. *Rev. Scient. Instrum.* **37**, 838–843 (1966).
25. J. W. DELLEUR, Flow direction measurement by hot-wire anemometry. *J. Eng. Mech. Div., Proc. A.S.C.E.* **92ii**, 45–70 (1966).
26. F. H. CHAMPAGNE, C. A. SLEICHER and O. H. WEHRMANN, Turbulence measurements with inclined hot-wires Part 1. Heat transfer experiments with inclined hot-wire. *J. Fluid Mech.* **28**, 153–175 (1967).
27. Y. AIHARA, D. R. KASSOY and P. A. LIBBY, Heat transfer from circular cylinders at low Reynolds numbers. II. Experimental results and comparison with theory. *Physics Fluids* **10**, 167–952 (1967).
28. B. NORMAN, Hot-wire anemometer calibration at high subsonic speeds. DISA Information No. 5, 5–19 (1967).
29. A. M. AHMAD, Forced convective heat tranfer to cooled cylinders at low Reynolds numbers and with large temperature differences. CARDE TR 588/68 (1968).
30. A. M. AHMAD, Comments on forced convective heat transfer to cooled cylinders. *Advances in Hot-Wire Anemometry*, pp. 65–70. Maryland Univ. (1968).
31. M. R. DAVIS and P. O. A. L. DAVIES, The physical characteristics of hot-wire anemometers. Univ. Southampton I.S.A.V.R. Tech. Rep. No. 2 (1968).
32. P. O. A. L. DAVIES and H. H. BRUUN, The performances of a yawed hot wire. Proc. Symp. on Instrumentation and Data Processing for Industrial Aerodynamics. N.P.L., Paper 10 (1968).
33. C. A. FRIEHE and W. H. SCHWARZ, Deviations from the cosine law for yawed cylindrical anemometer sensors. *J. Appl. Mech.* **35**, 655–662 (1968).
34. P. J. BOURKE, D. J. PULLING, L. E. GILL and W. H. DENTON, The measurement of turbulent velocity fluctuations and turbulent temperature fluctuations in the supercritical region by a hot wire anemometer and a 'cold' wire resistance thermometer. DISA Information No. 7, 3–12 (1969).
35. G. BACCAGLINI, D. R. KASSOY and P. A. LIBBY, Heat transfer to cylinders in nitrogen–helium and nitrogen–neon mixtures. *Physics Fluids* **12**, 1378–1381 (1969).

36. H. H. BRUUN, The performance of normal and yawed hot wires, Univ. Southampton I.S.V.R. Tech. Rep. No. 21 (1969).

37. H. HASSAN and J. C. DENT, Correction for temperature loading and high gas pressure effects for the constant temperature hot wire anemometer, *Br. J. Appl. Phys. (J. Phys. D) Ser.* 2. **2**, 85–91 (1969).

38. A. WHILLIER, An anemometer for the continuous measurement of air speed in mines, *Int. J. Rock Mech. Min. Sci.* **6**, 13–20 (1969).

39. B. KJELLSTROM and S. HEDBERG, Calibration of a DISA hot wire anemometer and measurements in a circular channel for confirmation of the calibration, DISA Information No. 9, 8–21 (1969).

40. J. WAY and P. A. LIBBY, Hot wire probes for measuring velocity and concentration in helium-air mixtures, *AIAA Jl* **8**, 976–978 (1970).

41. J. ULSAMER, Die Warmeabgabe eines Drahtes oder Rohres an einen senkrecht zur Achse strömenden Gasoder Flüssigkeitsstrom, *Forsch. Geb. Ing.* **3**, 94 (1932).

42. W. H. MCADAMS, *Heat Transmission*, third edition. Chap. X. McGraw-Hill, New York (1954).

43. H. KRAMERS, Heat transfer from spheres to flowing media, *Physica* **12**, 61 (1946).

44. E. R. G. ECKERT and E. SOEHNGEN, *Trans. Am. Soc. Mech. Engrs* **74**, 343–347 (1952).

45. M. D. SCADRON and I. WARSHAWSKY, Experimental determination of time constants and Nusselt numbers for bare-wire thermocouples in high velocity air streams, NACA TN 2599 (1952).

46. S. W. CHURCHILL and J. C. BRIER. *Chem. Engng Prog. Symp. Ser.* No. 17, **51**, 57 (1955).

47. W. J. M. DOUGLAS and S. W. CHURCHILL, Recorrelation of data for convective heat transfer between gases and single cylinders with large temperature differences, *Chem. Engng Prog. Symp. Ser.* No. 18, **52**, 23–28 (1956).

48. G. E. GLAWE and R. C. JOHNSON, Experimental study of heat transfer to small cylinders in a subsonic high-temperature gas stream, NACA TN 3934 (1957).

49. L. V. BALDWIN, V. A. SANDBORN and J. C. LAURENCE, Heat transfer from transverse and yawed cylinders in continuum, slip, and free molecular air flows, *J. Heat Transfer* **82**, 77–86 (1960).

50. J. C. LAURENCE and V. A. SANDBORN, Heat transfer from cylinders. ASME Symposium on Measurement in Unsteady Flow, 36–43 (1962).

51. P. D. RICHARDSON, Convection from heated wires at moderate and low Reynolds numbers, *AIAA Jl* **8**, 537–538 (1965).

52. J. KUX, Experimental methods in wind tunnels with special emphasis on the Hot-wire anemometer, AGARD Report 558 pt. 2 (1967).

53. D. BRADLEY and K. J. MATTHEWS, Measurement of high gas temperatures with fine wire thermocouples, *J. Mech. Engng Sci.* **10**, 299–305 (1968).

54. G. W. PRECKSHOT and J. W. GORMAN, Steady-state longitudinal and radial temperature distribution in internally heated finite wires, *Ind. Engng Chem.* **50**, 837–848 (1958).

55. R. BETCHOV, Nonlinear theory of a hot-wire anemometer, *Verh. Ned. Akad. Wet.* **52**, 195–207 (1949); (English Translation NACA TM 1346 (1952)).

56. M. HORVATIN. Some problems concerning the analytical evaluation of the characteristics of hot-wires immersed in a fluid of variable pressure and temperature, DISA Information No. 8, 23–31 (1969).

57. H. H. LOWELL, Design and application of hot-wire anemometers for steady state measurements of transonic and supersonic air speeds, NACA TN 2117 (1950).

58. L. S. G. KOVASZNAY, Turbulence measurements, *Physical Measurements in Gas Dynamics and Combustion*, edited by R. LADENBERG, Sec. F, Pt. 1, pp. 213–285. Princeton Univ. Press (1955).

59. T. VREBALOVICH, Heat loss and recovery temperature of fine wires in transonic transition flow, *Fifth Symposium on Rarefied Gas Dynamics*, edited by C. L. BRUNDIN, Vol. II, pp. 1205–1219. Academic Press, New York (1967).

60. D. C. COLLIS, Forced convection of heat from cylinders at low Reynolds numbers, *J. Aeronaut. Sci.* **23**, 697–698 (1956).

61. P. H. CHEVALIER, G. R. KUBANEK and W. H. GAUVIN, Heat transfer to cylinders in a confined jet at high temperature, *A.I.Ch.E. Jl* **16**, 520–527 (1970).

62. D. J. TRITTON, Experiments on the flow past a circular cylinder at low Reynolds numbers, *J. Fluid Mech.* **6**, 547 (1959).

63. F. S. SHERMAN, A survey of experimental results and methods for the transition regime of rarefied gas dynamics. *Third Symposium on Rarefied Gas Dynamics*, edited by J. A. LAURMANN, Vol. II, pp. 228–260. Academic Press, New York (1963).

64. W. B. BROOKS and G. E. REIS, Drag on a right circular cylinder at rarefied low speed-ratios. *ibid.* pp. 291–302.

65. E. A. BRUN, L. FACY and J. TROTEL, Etude experimentale de la trainee d'un reseau de fils dans un courant de gaz rarefile, *ibid.* pp. 303–316.

66. H. COUDERVILLE, P. TREPAND and E. A. BRUN. Drag measurements in slip and transition flow, *Fourth Symposium on Rarefied Gas Dynamics*, edited by J. H. DE LEEUW, Vol. I, pp. 446–466. Academic Press, New York (1965).

67. C. Y. LUI and R. PASSAMANECK, Kinetic theory description of flow over a cylinder at low speeds, *Fifth Symposium on Rarefied Gas Dynamics*, edited by C. L. BRUNDIN, Vol. I, pp. 607–622 Academic Press, New York (1967).

68. H. S. TSIEN, Superaerodynamics, mechanics of rarefied gases. *J. Aeronaut. Sci.* **13**, 653–664 (1964).

69. H. ATASSI and E. A. BRUN. Heat transfer from wires in a rarefied low speed flow, *Fifth Symposium on Rarefied Gas Dynamics*, edited by C. L. BRUNDIN, Vol. II, pp. 1221–1235. Academic Press, New York (1967).

70. H. C. LEVEY, Heat transfer in slip flow at low Reynolds numbers, *J. Fluid Mech.* **6**, 385–391 (1959).

71. D. R. KASSOY, Heat transfer from circular cylinders at low Reynolds numbers. I. Theory for variable property flow *Physics Fluids* **10**, 938–946 (1967).

72. G. N. PATTERSON. A synthetic view of the mechanics of rarefied gases, *AIAA Jl* **3**, 577–590 (1965).

73. S. A. SCHAAF and P. L. CHAMBRE, *Flow of Rarefied Gases*. Princeton Aeronautical Paperbacks No. 8. Princeton Univ. Press, Princeton N.J. (1961).

74. S. A. SCHAAF and L. TALBOT. Mechanics of rarefied gases, NAVORD Rep. 1488 (Vol. 5), Handbook of Supersonic Aerodynamics Sec. 16, Bureau of Ordnance, Washington (1959).

75. C. FORBES DEWEY. A correlation of convective heat transfer and recovery temperature data for cylinders in compressible flow. *Int. J. Heat Mass Transfer* **8**, 245–252 (1965).

76. H. J. BUMELBURG. Heat loss from very thin heated wires in rarefied gases, *Physics Fluids* **2**, 717–718 (1959).

77. L. LEES and C-Y. LIU, Kinetic-theory description of conductive heat transfer from a fine wire. *Physics Fluids* **5**, 1137–1148 (1962).

78. G. S. SPRINGER and R. RATONYI, Heat conduction from circular cylinders in rarefied gases. *J. Heat Transfer* **87C**, 493–498 (1965).

79. F. C. HURLBUT, Note on conductive heat transfer from a fine wire. *Physics Fluids* **7**, 904–906 (1964).

80. F. M. SAUER and R. M. DRAKE, Forced convection heat transfer from horizontal cylinders in a rarefied gas. *J. Aeronaut. Sci.* **20**, 175–180 (1953).

81. E. H. KENNARD, *Kinetic Theory of Gases*. McGraw-Hill, New York (1938).

82. R. G. DEISSLER, Analysis of second-order slip flow and temperature jump boundary conditions for rarefied gases. *Int. J. Heat Mass Transfer* **7**, 681–694 (1964).

83. H. MIKAMI, Y. ENDO and Y. TAKASHIMA, Heat transfer from a sphere to rarefied gas mixtures, *Int. J. Heat Mass Transfer* **9**, 1435–1448 (1966).

84. L. L. KAVANAU, Heat transfer from spheres to a rarefied gas in subsonic flow. *Trans. Am. Soc. Mech. Engrs* **77**, 617–623 (1955).

85. I. EASTERMANN, Gases at low density, *High Speed Aerodynamics and Jet Propulsion*. Vol. I. *Thermodynamics and Physics of Matter*. edited by D. ROSSINI. p. 736. Princeton Univ. Press. Princeton N.J. (1955).

86. H. Y. WACHMAN, Thermal accommodation coefficient: A critical survey, *ARS Jl* **32**, 2–12 (1962).

87. J. P. HARTNETT, A survey of thermal accommodation coefficients. *Advances in Applied Mechanics*. Suppl. 1. *Rarefied Gas Dynamics*, edited by L. TALBOT, pp. 1–28. Academic Press, New York (1961).

88. M. KAMINSKY, *Atomic and Ionic Impact Phenomena on metal surfaces*. Chap. 1. Springer, Berlin (1965).

89. F. M. DEVIENNE, Low density heat transfer, *Advances in Heat Transfer*, Vol. 2, edited by J. P. HARTNETT and T. F. IRVINE, pp. 271–365. Academic Press, New York (1965).

90. L. TRILLING, Thermal accommodation of rare gases on clean metal surfaces—comparison of a simplified classical lattice theory with experiments, *Fifth Symposium on Rarefied Gas Dynamics*, edited by C. L. BRUNDIN, pp. 139–154. Academic Press, New York (1967).

91. L. B. THOMAS, A collection of some controlled surface thermal accommodation coefficient measurements. *ibid.* 155–162.

92. E. M. SCHMIDT and R. J. CRESCI. Hot-wire anemometry in low density flows. AIAA paper No. 70–589 (1970).

93. G. L. DE POORTER and A. W. SEARLY. Energy exchange between a hot tungsten surface and cold gases. *J. Chem. Phys.* **39**, 925 (1963).

94. H. VON UBISCH, On the conduction of heat in rarefied gases and its manometric application. *Appl. Sci. Res.* **A2**, 364 (1949–1950).

95. I. AMDUR and L. A. GUILDNER. Thermal accommodation coefficients on gas-covered tungsten, nickel and platinum, *J. Am. Chem. Soc.* **79**, 311–315 (1957).

96. H. Y. WACHMAN, Accommodation coefficients of nitrogen and helium on nitrogen covered tungsten between 325°–496 K. *J. Chem. Phys.* **45**. 1532 (1966).

97. B. G. DICKENS, The effect of accommodation on heat conduction through gases. *Proc. R. Soc.* **143A**, 517 (1934).

98. E. R. GRILLY, W. J. TAYLOR and J. L. JOHNSTON. Accommodation coefficient for heat conduction between gas and bright platinum for nine gases between 80°K (or their boiling points) and 380°K. *J. Chem. Phys.* **14**, 435–440 (1946).

99. M. L. WIEDMANN and P. R. TRUMPLER. Thermal accommodation coefficients, *Trans. Am. Soc. Mech. Engrs* **68**, 57–64 (1946).

100. W. J. TAYLOR and H. L. JOHNSTON, An improved hot wire cell for accurate measurements of thermal conductivities of gases over a wide temperature range. *J. Chem. Phys.* **14**, 219–233 (1946).

101. K. SCHÄFER, Energieübertragungsmechanismus und Reaktionsgeschwindigkeit an metallischen Oberflächen, *Z. Electrochem.* **56**, 398–403 (1952).

102. L. B. THOMAS and R. C. GOLICKE. A comparative study by the temperature jump and low pressure methods and thermal conductivities of He. Ne. CO_2. *J. Chem. Phys.* **22**, 300 (1954).

103. D. V. ROACH and L. B. THOMAS. Determination of the thermal accommodation coefficients of gases on clean surfaces at temperatures above 300°K by the temperature jump method. *Fifth Symposium on Rarefied Gas Dynamics*, edited by C. L. BRUNDIN, Vol. 1. pp. 163–172. Academic Press, New York (1967).

104. R. N. OLIVER and M. FARBOR. Experimental determination of accommodation coefficients as functions of temperature for several metals and gases. Calif. Inst. Tech., Jet Propulsion Lab.. Mem. 9–19 (1950).

105. F. O. GOODMAN and H. Y. WACHMAN, Formula for thermal accommodation coefficients, *J. Chem. Phys.* **46**, 2376–2386 (1967).

106. E. R. G. ECKERT and R. M. DRAKE, *Heat and Mass Transfer*. p. 504. McGraw-Hill, New York (1959).

107. E. M. WISE and R. F. VINES, *The Platinum Metals and Their Alloys*. Int. Nickel Co., New York (1941).

108. S. GOLDSTEIN, *Modern Developments in Fluid Mechanics*. Vol. 1, p. 299, Oxford (1943).

109. X. J. R. HVULA, Analysis of suddenly started laminar flow in the entrance region of a circular tube, *Appl. Sci. Res.* **A21**, 248 (1969).

110. J. TOPPING, *Errors of Observation and Their Treatment*. Chapman & Hall, London (1955).

111. J. PICHON, Comparison of some methods of calibrating hot-film probes in water, DISA Information No. 10, 15–21 (1970).

112. R. P. DRING and B. GEBHART, Hot-wire anemometer calibration for measurements at very low velocity, *J. Heat Transfer* **91**, 241–244 (1969).

113. S. B. REED and M. P. SPRANGE, Flow meter calibration for any gas in the range 1–500 litres per hour. *I/EC Fundamentals* **7**, 651–655 (1968).

114. DISA, The low velocity anemometer, DISA Information No. 7, p. 32–35 (1969).

115. I. PROUDMAN and J. K. A. PEARSON, Expansion at small Reynolds numbers for the flow past a sphere and a circular cylinder, *J. Fluid Mech.* **2**, 237–262 (1957).

116. S. K. FRIEDLANDER, Mass and heat transfer to single spheres and cylinders at low Reynolds numbers, *A.I.Ch.E.Jl.* **3**, 43–48 (1957).

117. S. C. R. DENNIS, J. D. HUDSON and N. SMITH, Steady laminar forced convection from a circular cylinder at low Reynolds numbers, *Physics Fluids* **11**, 933–940 (1968).

118. P. F. HODNETT, Slow compressible flow past a circular cylinder, *Physics Fluids* **11**, 1636–1647 (1968).

119. W. W. WOOD, Calculations for anemometry with fine hot wires, *J. Fluid Mech.* **32**, 9–19 (1968).

120. C. A. HIEBER and B. GEBHART, Low Reynolds number heat transfer from a circular cylinder, *J. Fluid Mech.* **32**, 21–28 (1968).

121. P. F. HODNETT, Low Reynolds number flow of a variable property gas past a heated circular cylinder, *J. Fluid Mech.* **39**, 465–476 (1969).

122. P. F. HODNETT, Low Reynolds number flow past a heated circular cylinder, *Physics Fluids* **13**, 2429–2431 (1970).

123. K. TAMADA and K. YAMAMOTO, Flow of rarefied gas past a circular cylinder at low Mach number, *Mem. Fac. Engng Kyoto Univ.* **30**, 132–152 (1968).

124. K. YAMAMOTO, Heat transfer from a circular cylinder in a rarefied gas flow, *J. Phys. Soc., Japan* **28**, 783–789 (1970).

125. B. GEBHART, T. AUDUNSON and L. PERA, Forced, mixed and natural convection from long horizontal wires, experiments at various Prandtl numbers, Fourth International Heat Transfer Conference, Paper NC 3–2, Paris (1970).

126. R. A. SVEHLA, Estimated viscosities and thermal conductivities of gases at high temoerature, NASA TR R–132 (1962).

127. *International Critical Tables*. McGraw-Hill, New York (1929).

128. C. R. WILKIE, A viscosity equation for gas mixtures, *J. Chem. Phys.* **18**, 517 (1950).

129. E. A. MASON and S. C. SAXENA, Approximate formulae for the thermal conductivity of gas mixtures, *Physics Fluids* **1**, 361 (1958).

130. B. GEBHART and L. PERA, Mixed convection from long horizontal cylinders, *J. Fluid Mech.* **45**, 49–64 (1970).

ETALONNAGE D'UN ANEMOMETRE A FIL CHAUD POUR LA MESURE DES FAIBLES VITESSES DE GAZ

Résumé—Des fils chauds ont été étalonnés dans le domaine $0 < Re < 20$ pour des valeurs significatives de Kn atteignant 0,12 différentes températures de chauffage et différents rapports de forme du fil. Les étalonnages ont été effectués dans une soufflerie et dans une enceinte à pression variable dans laquelle la composition du gaz pouvait être modifiée . Les influences des différents paramètres sont dégagées et discutées. La relation recommandée est :

$$N_c = 0{,}34 + 0{,}65\, Re^{0{,}45}$$

où

$$N_c = \frac{N}{1 - 2\,KnN}$$

pour $1/d > 400$, $0{\cdot}02 < Re < 20$ et avec les valeurs des propriétés prises pour la température moyenne du gaz. On discute l'importance d'une connaissance précise du coefficient d'accommodation thermique à des valeurs plus grandes de Kn.

EICHUNG VON HITZDRAHTANEMOMETERN ZUR MESSUNG KLEINER GASGESCHWINDIGKEITEN

Zusammenfassung—Es wurden Hitzdrähte im Bereich $0 < Re < 20$ bei ausgezeichneten Werten von Kn bis 0,12. verschiedenen Temperaturbelastungen und Durchmesser-Längen-Verhältnissen des Drahtes geeicht. Die Eichungen wurden in einem Windkanal und in einer Anordnung für veränderlichen Druck

durchgeführt. in der auch die Gaszusammensetzung geändert werden konnte. Die Einflüsse der verschiedenen Veränderlichen werden gezeigt und erörtert.
Das empfohlene Eichgesetz lautet:

$$N_c = 0.34 + 0.65\,Re^{0.45}$$

wobei $N_c = (N/1 - 2\,Kn\,N)$ bei $e/d > 400$. $0.02 < Re < 20$ und den Stoffeigenschaften bei der mittleren Gastemperatur. Die Bedeutung einer genauen Kenntnis des thermischen Anpassungskoeffizienten bei grösseren Werten von Kn wird erörtert.

ТАРИРОВКА ТЕРМОАНЕМОМЕТРА ДЛЯ ИЗМЕРЕНИЯ МАЛЫХ СКОРОСТЕЙ ГАЗА

Аннотация—Термоанемометры тарировались в режиме $0 < Re < 20$ при числах Кнудсена до 0,12 при различных тепловых нагрузках и различных соотношениях длины проволочки и диаметра. Тарировка проводилась в аэродинамической трубе с помощью манометра, при этом мог изменяться состав. Показано и анализируется влияние различных воздействий на тарировку. Предлагается следующее выражение для тарировки

$$N_c = 0{,}34 + 0{,}65\,Re^{0{,}45}$$

где
$$N_c = N/1 - 2\,Kn\,N$$
при
$$l/d > 400,\ 0{,}02 < Re < 20$$

для значений свойств, взятых при средней температуре газа. Рассматривается необходимость знания точного значения коэффициента тепловой аккомодации при больших числах Kn.